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Maintenance Policy for a Repairable System with a Linearly Increasing Hazard rate and Predetermined Availability

Mwaba Coster[1]Qiong Liu[2]
[1,2]*State Key Laboratoryof Digital Manufacturing Equipment & Technology*
School of Mechanical Science and Engineering, Huazhong University of Science and Technology
1037Luo Yu Road, Wuhan, Hubei 430074, P.R. China

***Abstract:* -** This paper discusses the determination of an optimal maintenance policy for a repairable system exhibiting a linearly increasing hazard rate, a constant repair rate and a predetermined availability. Under the preventive maintenance policy, the system availability is guaranteed and the time interval between preventive maintenance actions incurring minimal cost is analysed and presented.

Keywords: *Preventive maintenance,Corrective maintenance, Linear hazard rate, Availability*

I. INTRODUCTION

Repairable systems assume a whole range of performance levels ranging from perfect functioning to complete failure. In practice, repairable systems deteriorate because of age and cumulative wear, and perfect functioning is not always guaranteed. As a result, Preventive and Corrective maintenance actions or system replacement are introduced in order to attain the desirable availability and reliability of such systems. With complex and multi-unit systems, maintenance optimization is playing an increasing role in maximizing plant availability and equipment effectiveness while minimizing equipment life cycle cost.

Many authors have proposed several preventive maintenance models, either periodic or sequential, and obtained PM policies by optimizing several criteria regarding the optimal maintenance actions and operating cost. In[1] the author observed that all maintenance actions aim to improve the system reliability, whereas preventive maintenance modelling focuses on the cost optimization criterion, ignoring the system reliability requirements. A block renewal model was proposed in [2] where imperfect repairs could be performed between renewals. The model determines the renewal interval to minimise the rate of change in cost per unit time considering the renewal cost, the repair cost and the cumulative hazard rate function for a Weibull distribution. An opportunistic maintenance model for the increasing failure rate (IFR) of components in a repairable system was presented in [3]. They analysed the failure behaviour of the components between two overhauls. Four preventive maintenance approaches were proposed by [4], this included a group maintenance model to balance the unplanned failures and PM costs through the analysis of PM intervals which would limit the probability of system failure to a predefined maximum tolerable level. In [5] a hybrid age-based model for imperfect PM involving maintainable and non-maintainable failure modes was developed. They determined the number of PM actions and the length of PM intervals that minimize the total long-term expected cost per unit time. More comprehensive discussions on PM from both a theoretical and application point of view can be found in the following literature: [6],[7], [8],[9], [10], [11],[12].

Recently, the risk based and reliability based approaches to maintenance optimization have been addressed by many researchers. Risk based maintenance optimization analyses the effect on the main objectives of alternative sets of strategies, whereas, reliability models looks at the reduction of systems probability of failure i.e., availability or the degradation level of components. The authors in [13] presented availability centred preventive maintenance by simultaneously considering three actions, mechanical service, repair and replacement for a multi component system based on availability. A new reliability based optimal maintenance scheduling method based on the ordering list of maintenance effects was proposed in [14].

The method is realised by increasing the number of maintenance tasks of some elements that possess high maintenance effects in order to increase the system availability, while decreasing the number of maintenance tasks of some other elements that possesslow maintenance effects in order to reduce the number of maintenance tasks. [15]presented a methodology for the design of an optimum inspection and maintenance program. The methodology is based on integrating a reliability approach and a risk assessment strategy to obtain an optimum maintenance schedule.In [16] using renewal theory, the transient behaviour of a multi-unit system was studied. The instantaneous availability for a finite time was analyzed and a methodology was developed to find time intervals to preventive and opportunistic maintenance such that cost is minimized. His components had binary levels of degradation, i.e. operating or failed. In [17] theauthors discussed the instantaneous availability for two models, when the perfect repair or replacement time is a constant or random variable. They provided recursive equations for the instantaneous availabilities of the two models. The effect of reliability, budget and breakdown outage cost on the analysis of optimal maintenance intervals was analysed in[18]. Three models were proposed to calculate optimal maintenance intervals for a multi-component system subjected to minimum required reliability, maximum allowable budget, and minimum total cost

Authors in [19] determined component maintenance priorities in a series-parallel system. The optimal maintenance periods of these components are determined to minimize total maintenance cost, given the allowable worst reliability of a repairable system. [20]investigated a bi-objective imperfect PM model of a series-parallel system. They developed a unit-cost cumulative reliability expectation measure to evaluate the extent to which maintainingeach individual component benefits the total maintenance cost and system reliability over the operationallifetime.

Most of the PM studies of repairable systems consider cost based or reliability analysis approach based on operation data. It was pointed out in [21] that, maintenance optimization should not start with developing a maintenance model and try to fit an application to it, but it should start with an application and try to fit a maintenance optimization model to it.

This paper analyses the determination of an optimal preventive maintenance policy for a repairable system when a combination of both availability and cost are used as performance measures. The system is assumed to exhibit a linearly increasing hazard rate and a constant repair rate. The objective of this analysis is to determine the preventive maintenance frequency resulting in the minimum maintenance cost per unit time to maintain the desired system availability.

II. MODEL DESCRIPTION AND ASSUMPTIONS

This paper considers two maintenance policies for a repairable system:

a. Policy 1: No preventive maintenance (PM) action is performed on the system.As a corrective maintenance (CM) action, it is replaced by a new one and restored to ‘as good as new’ condition. The time taken to replace a failed system is a random variable with a constant mean time.

b. Policy 2: The system is periodically replaced by new one at set times $t = kT$, k = 1, 2, … . Any failure between replacement times is minimally repaired through corrective maintenance (CM), and as in policy 1, time taken is a random variable. Minimal repair does not change the hazard rate of the system prior to failure. The time taken for minimal repair is negligible.

And the following notations will be used:

λ: failure rate
μ: repair rate
h(t): hazard function
H(t): cumulative hazard function
$\dot{A}$: predetermined availability
$\ddot{A}$: availability incurring minimum maintenance cost
A: limiting availability
C_p: cost of PM action replacing system by a new one
C_r: cost of CM through minimal repair
$\dot{t}$: time interval between PM actions guaranteeing predetermined availability
$\dot{T}$: time interval incurring minimum maintenance cost per unit time
C(T): maintenance cost per unit time over the time interval (0, T)

The mathematical model of the system provides a useful tool for deriving the expressions for the system performance characteristics. A numerical example is given to demonstrate how the cost based measure and predetermined availability can be used as a basis to determine optimal maintenance policies for a repairable system.

III. LINEARLY INCREASING HAZARD RATE AND AVAILABILITY

Repairable systems degrade during the course of their operational life and are more likely to follow a distribution with a strictly increasing hazard function. Due to its flexibility, the Weibull distribution is the most widely used failure distribution in reliability applications. For the model under consideration, the hazard function of the Weibull distribution has the form

$$h_{(t)} = \alpha\beta^{\alpha}t^{\alpha-1} \tag{1}$$

Where $\alpha > 0$ and $\beta > 0$ are the shape and scale parameters respectively, for all $t \geq 0$. The mean of the distribution is

$$\frac{1}{\beta}\Gamma\left(1+\frac{1}{\alpha}\right) = \frac{1}{\alpha\beta}\Gamma\left(\frac{1}{\alpha}\right) \tag{2}$$

Where $\Gamma(\cdot)$ is the Gamma function. The hazard function in (1) is more appropriate for modelling and analysis of life distributions with constant (α = 1), strictly increasing (α> 1) and strictly decreasing (α < 1). This paper considers a special case of the Weibull distribution when α = 2. This case is commonly known as the Rayleigh distribution whose hazard function is a line with slope $2\beta^2 t$[22], i.e.,

$$h(t) = 2\beta^2 t \quad (3)$$

Therefore, a system that exhibits a linearly increasing hazard function has a lifetime distribution which obeys the Rayleigh distribution. Substituting 2 for α in equation (2), the mean Rayleigh distribution becomes

$$\frac{1}{2\beta}\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2\beta} \quad (4)$$

With an assumption that a system has a constant failure rate λ and constant repair rate μ, and that, the operating and repair time distributions are arbitrary continuous distributions with respective means, $\frac{1}{\lambda} and \frac{1}{\mu}$, then from the theory of alternating renewal processes[23], the Availability A is,

$$A = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}+\frac{1}{\mu}} = \frac{\mu}{\mu+\lambda} \quad (5)$$

Application of policy 1 results in the system availability with a lifetime distribution which obeys the Rayleigh distribution whose constant repair rate is μ,

$$A_1 = \frac{\frac{\sqrt{\pi}}{2\beta}}{\frac{\sqrt{\pi}}{2\beta}+\frac{1}{\mu}} = \frac{\mu\sqrt{\pi}}{\mu\sqrt{\pi}+2\beta} \quad (6)$$

Equation (5) shows that, the system availability is a decreasing function of the hazard rate and an increasing function of the repair rate. Therefore, to maintain a specific value of availability$\dot{A}$, with known constant repair rate,μ, the constant hazard rate must satisfy the relationship

$$\lambda = \frac{\mu(1-\dot{A})}{\dot{A}} \quad (7)$$

IV. DETERMINATION OF PREVENTIVE MAINTENANCE INTERVALS CONSIDERING AVAILABILITY

When the predetermined Availability $\dot{A}$, of the system is not achieved with Policy 1, i.e. without Preventive maintenance, then performing preventive maintenance actions should be taken into consideration (Policy 2). However, before PM actions are performed, the predetermined availability$\dot{A}$, must be larger than the availability A_1 in equation (6) as the system could have achieved this without PM actions. The desired constant hazard rate of equation (7) should be set less than that of equation (4). The predetermined availability $\dot{A}$,should satisfy the inequality

$$\frac{\mu(1-\dot{A})}{\dot{A}} < \frac{2\beta}{\sqrt{\pi}} \Rightarrow \dot{A} > \frac{\mu\sqrt{\pi}}{2\beta+\mu\sqrt{\pi}} \quad (8)$$

Once policy 2 is chosen, the time interval between preventive maintenance actions should be determined. The determination of the time interval during which the linearly increasing hazard rate is substituted by a constant hazard rate while maintaining a predetermined availability level is obtained following the proposal by [8].

The cumulative hazard function corresponding to the linearly increasing hazard rate of equation (3) over the time interval (0, t) is

$$H(t) = \int_0^t h(t)\,dt = \beta^2 t^2 \quad (9)$$

The mean hazard rate based on (9) during the interval (0, t) becomes

$$\frac{H(t)}{t} = \frac{1}{t}\int_0^t h(t)dt = \beta^2\, t \quad (10)$$

Equating equations (10) and (7) gives the time interval $\dot{t}$, between consecutive preventive maintenance actions

$$\beta^2 t = \frac{\mu(1-\dot{A})}{\dot{A}} \Rightarrow \dot{t} = \frac{\mu(1-\dot{A})}{\beta^2\dot{A}} \quad (11)$$

This shows that, for the predetermined availability in policy 2 to be maintained, the time interval between two maintenance actions should be equal or less than $\dot{t}$.

V. DETERMINATION OF PREVENTIVE MAINTENANCE INTERVALS CONSIDERING AVAILABILITY AND MAINTENANCE COST

When the predetermined availability,$\dot{A}$, of the system is not achieved with Policy 1, i.e.,$A_1 < \dot{A}$ then the periodic maintenance policy 2 should be considered, as earlier stated. The system availability and maintenance cost will be considered simultaneously. The periodic maintenance actions will be conducted at multiples of the cycle length T, i.e., $t = kT$, k = 1, 2, ….

The cycle cost is the sum of preventive replacement cost at the end of the cycle and the cost of corrective repairs during the cycle. Therefore, the maintenance cost per unit time is

$$C(T) = \frac{C_p + C_r \int_0^T 2\beta^2 t dt}{T}$$

$$= \frac{C_p}{T} + C_r \beta^2 T \tag{12}$$

Setting $\frac{dC(T)}{dT} = 0$, yields the time interval with the minimum maintenance cost per unit time:

$$\dot{T} = \frac{1}{\beta}\sqrt{\frac{C_p}{C_r}} \tag{13}$$

And its corresponding cost becomes $C(\dot{T}) = 2\beta\sqrt{C_p C_r}$

If $\dot{T} \leq \dot{t}$ then the optimal time interval between preventive maintenance actions is $\dot{T}$.

From equation (13), the system availability would now be

$$\ddot{A} = \frac{\mu}{\mu + \beta^2 \dot{T}} = \frac{\mu}{\mu + \beta\sqrt{C_p/C_r}} \tag{14}$$

If $\dot{T} > \dot{t}$, then determination of the time interval between preventive maintenance actions becomes a problem of choice as the predetermined availability and the minimum maintenance cost per unit time can not be satisfied at the same time. Preference between them should be decided depending on the prevailing operating conditions.

When the availability is preferred to be kept constant,i.e.$\dot{t}$ in equation (11) would be the best choice because it guarantees the predetermined availability $\dot{A}$ resulting in minimal maintenance cost per unit time over the interval (0, $\dot{t}$),

$$C(\dot{t}) = \frac{C_p}{\dot{t}} + C_r \beta^2 \dot{t} \tag{15}$$

If the maintenance cost per unit time is preferred to be minimised, the time interval between preventive maintenance actions,$\dot{T}$,is gotten from equation (13). This gives availability $\ddot{A}$from equation (14) which is less than the predetermined availability $\dot{A}$ and the system availability should satisfy the inequality (16)

$$\ddot{A} > \frac{\mu\sqrt{\pi}}{2\beta + \mu\sqrt{\pi}} \tag{16}$$

Otherwise, maintaining minimal cost implies that the limiting availability is given up. This is due to the fact that it is greater than the system availability and can be achieved without PM action.

VI. NUMERICAL EXAMPLE

Numerical example is presented on the decision making process considering possible cases which determines the optimum maintenance policy guaranteeing the predetermined availability.

A system exhibits a linearly increasing hazard rate, $h(t) = 2 \times 10^{-6} t$ (t is in hours), and a constant repair rate $\mu = 2 \times 10^{-2}$. The predetermined availability is desired to be at least, $\dot{A}$= 98%. The cost of PM action replacing a new system by a new one is (C_p) is \$4000 and the cost of corrective maintenance action through minimal repair (C_r) is \$200.

From equations (3) and (6), the corresponding limiting availability is A = 95%, which is less than the predetermined $\dot{A}$= 98%. In order to maintain the predetermined availability, the longest time interval between PM actions is $\dot{t}$ = 408 hours obtained from equation (11). However, the time interval incurring the minimum maintenance cost per unit time $\dot{T}$ = 4472 hours obtained from (13) is much longer than $\dot{t}$ = 408 and results in a system availability of $\ddot{A}$ = 82% from equation (14). The results are shown in fig.1.

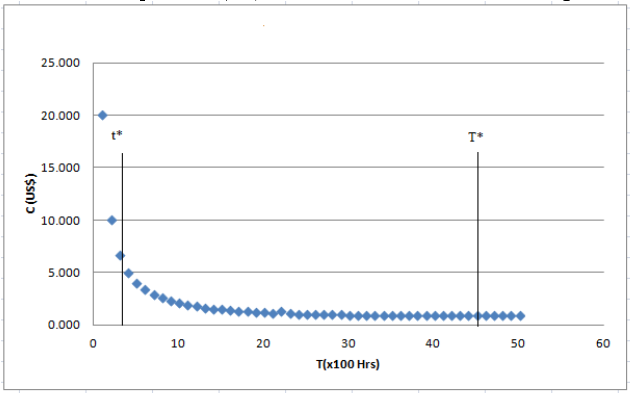

Fig.1 Maintenance cost per unit time

Depending on the prevailing operating conditions,maintenance decision making is based on the following two alternatives, either availability or minimum cost:

a. Choosing the minimum maintenance cost per unit time of \$1.0 evaluated from equation (12) and giving up the predetermined availability of $\dot{A}$= 98%. This would ensure that the system availability is kept at $\ddot{A} =$ 82% by taking $\dot{T} = 4472$ hours as the time interval between preventive maintenance actions.
b. Maintaining the predetermined availability of $\dot{A}$= 98% and give up the minimum cost. This option adopts $\dot{t} = 408$ hours as the time interval between maintenance actions, but the lowest cost of maintaining the system over the interval $(0,\dot{t})$ would be \$10, from equation (15).

VII. CONCLUSION

This paper proposed a preventive maintenance model for a system exhibiting a linearly increasing hazard rate and a constant repair rate.The combination of both system predetermined availability and maintenance costs are considered as performance measures. It is shown that a system showing such behaviour follows the Rayleigh lifetime distribution, and when availability is considered as a performance criteria, the system's limiting availability can be calculated using the mean lifetime of the distribution and mean repair time. The maintenance intervals are analysed depending on which performance measure is desired. The problem of interest was to determine the time interval between preventive maintenance actions when a combination of both cost and availability is used as measure of system performance. A case arises where the time interval incurring minimum maintenance cost does not guarantee the predetermined availability. Another case is where the time interval incurring the minimum maintenance cost guarantees the predetermined availability. In both instances, the guidelines for determining the maintenance policy are presented.

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